Statistical Improvements in the Unmanned Spacecraft
Cost Model - Monograph #1

by

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I. INTRODUCTION

A. Purpose

The purpose of this monograph is to test some of the CER's in the Unmanned Spacecraft Cost Model for non-constant error variances, or heteroskedasticity, and to take corrective statistical action if and where the problem is found.

B. Background

The Unmanned Spacecraft Cost Model is a set of regression equations or Cost Estimating Relationships (CER's) designed to explain the costs of spacecraft subsystems, such as electrical power supplies, apogee kick motors, and communication electronics. Technical and performance characteristics are used to explain costs, with the model based on 35 military, communications, weather, experimental, and lunar-probe spacecraft.

The model presents equations for explaining both first-unit recurring costs and total nonrecurring costs, using "normalized" and "unnormalized" data. Normalized data are costs adjusted for "technology carryover" and "complexity of design," with these terms accounting for the impact on cost of technological change and hardware sophistication. Unnormalized data, on the other hand, are costs in deflated but otherwise raw form.

Equations of the model, both normalized and unnormalized, are presently estimated independently of one another, using ordinary least squares (OLS) or nonlinear regression. Based on theoretical grounds, however, several improvements to the model may result from:

- Testing equations for heteroskedasticity, and taking corrective action, if necessary
- Estimating power-function regression equations using Goldberger's unbiased estimator [1] rather than OLS
- Investigating alternative specifications of single equations
- Determining the proper form of the random error term in each equation, e.g., additive or multiplicative, and then using this specification to drive the estimation technique
- Estimating total spacecraft unit cost as a system of simultaneous equations

C. Scope

This paper, the first of five statistical monographs on the spacecraft model, is limited to the first area of research, i.e.,

testing equations for heteroskedasticity. And while no effort is made to gather cost, technical and performance data on recently built satellites, the points illuminated here should be applicable to future model-building efforts.

II. TESTS FOR HETEROSKEDASTICITY

A. Explanation

A crucial assumption in regression analysis is that the spread of observations on a dependent variable around a population regression line is invariant with respect to changes in the value of an explanatory variable. Put another way, the variance of an equation's error term should be constant from one observation to another. When it isn't, the errors are called heteroskedastic, and OLS standard errors are biased. Figure 1 illustrates the problem.

Heteroskedasticity in the spacecraft model, if present, could take either of two forms, at least in theory. First, the variance of unit costs might increase in proportion to the value of an explanatory variable such as subsystem weight. If the mean cost of a heavy system is a lot higher than the mean cost of a light one, for example, then the magnitude of the delta between the two costs may imply different variances.[1]

On the other hand, however, the opposite case may hold. Namely, the unit costs of lightweight systems might be more volatile than those of heavyweight systems due to:

- Rapid technological change in the aerospace industry in the early and mid 1960's when many of the lightweight

systems were built, thus inducing a large variance in costs.

- Efforts in some cases to pack a lot of technical performance into a lightweight package, thus driving costs above the norm.

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[1] Let the mean cost of a lightweight Apogee Kick Motor (AKM) equal \$100, and let the mean cost of a heavy one equal \$1000. Next, assume that three values are observed, with identical spreads of ±10% about the mean in each case:

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 110,100,90    for the light AKM
1100,1000,900  for the heavy AKM
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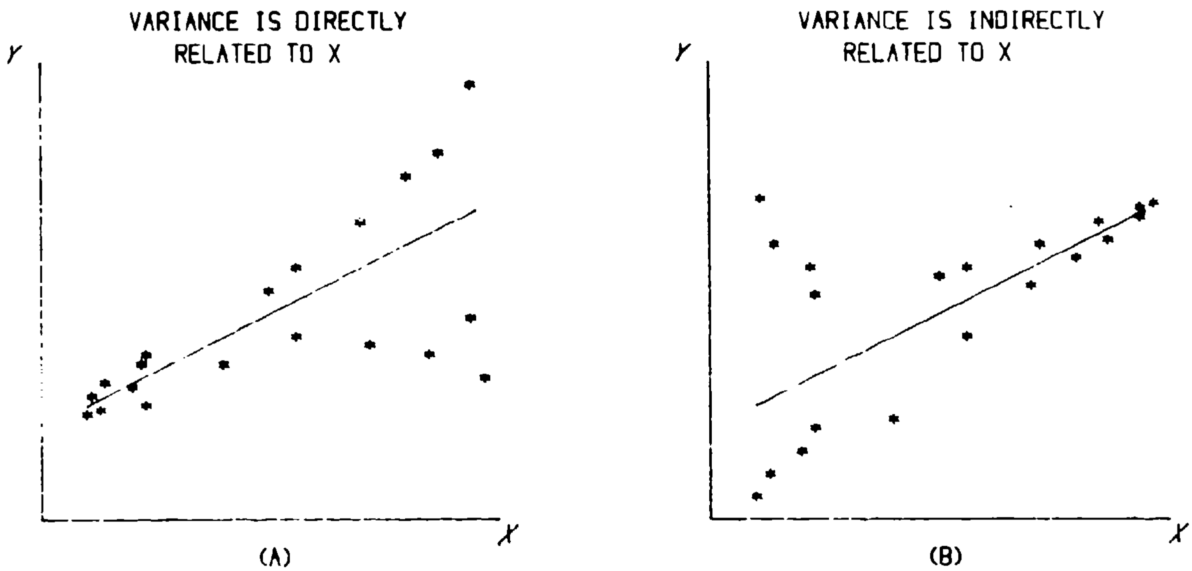
The sample variance is 100 in the first case but 10,000 in the second.

EXAMPLES OF HETEROSKEDASTICITY



In each of these graphs the dots represent ordered pairs of observations on Y and X, the dependent and explanatory variables in a simple linear relation. The lines represent population regression equations, which are almost always unknown. The vertical distance between a dot and a line is an observation on the error term.

Heteroskedasticity occurs when the variance of the regression equation's error term is not constant. In graph (a) the variance increases as values of X increase. In graph (b), on the other hand, an inverse relationship holds.

FIGURE 1

B. Tests

Park's test is used to determine which form of heteroskedasticity, if either, is present in the spacecraft model. The test, detailed in Appendix 1, is performed on all first-unit recurring cost CER's which are based on unnormalized data and for which a reasonable number of degrees of freedom is available.[2] The null hypothesis in all cases is that an equation's error term is homoskedastic. The alternative hypothesis is that the error variance is related, either directly or inversely, to the magnitude of the explanatory variable.

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[2] To limit the scope of this study to manageable size, two classes of CER's were not tested for heteroskedasticity

- Equations for estimating non-recurring costs
- Equations based on normalized data.

Further, the test was not performed on subsystems with a paucity of observations

- Apogee Kick Motor for 1-Axis Satellites (sample size of 5)
- Apogee Kick Motor for 3-Axis Satellites (sample size of 6)
- Dispenser (sample size of 4).

Finally, inherently nonlinear equations of the model were estimated in power-function form, i.e.,

$$Y = \alpha + X^{\beta} + \varepsilon \quad \text{as} \quad Y = \alpha X^{\beta} e^{\varepsilon}.$$

And linear equations with Y-intercepts restricted to zero were estimated in unrestricted form, i.e.,

$$Y = \beta X + \varepsilon \quad \text{as} \quad Y = \alpha + \beta X + \varepsilon .$$

As Table 1 shows, the null hypothesis of homoskedasticity is rejected for three of the sixteen equations examined

(1) Attitude Control

(2) Attitude and Reaction Control

(3) Program Level

And as Figures 2 through 4 illustrate, the spread of regression residuals is inversely related to the magnitude of X in the first two CER's, and directly related in the last.

TABLE 1

RESULTS OF THE PARK TEST FOR HETEROSKEDASTICITY
(Unit-Cost Equations Based on Unnormalized Data)

Equation	Sample Size	t-Statistic
Structure, Thermal Control, and Interstage	31	0.807
Telemetry, Tracking, and Command (TT&C)	29	-1.557
Communications	15	-0.197
Communications Antennas	12	-2.199
Communications Electronics	12	1.016
Combined Communications and TT&C	15	-0.403
Attitude Control	30	-2.373
Attitude Determination	16	-0.398
Attitude and Reaction Control	16	-3.158
Power Supply (subsynchronous altitude)	11	-0.660
Power Supply (synchronous altitude)	19	-0.185
Platform (without mission equipment)	31	-1.156
Program Level (as a function of platform)	30	2.521
Program Level (communications satellites)	15	1.839
LOOS (for satellites with an AKM)	12	1.804
LOOS (for satellites without an AKM)	10	-0.351

NOTE: Figures underlined represent cases where the null hypothesis of homoskedasticity is rejected at the 5% level of significance using the two-tailed t-test.

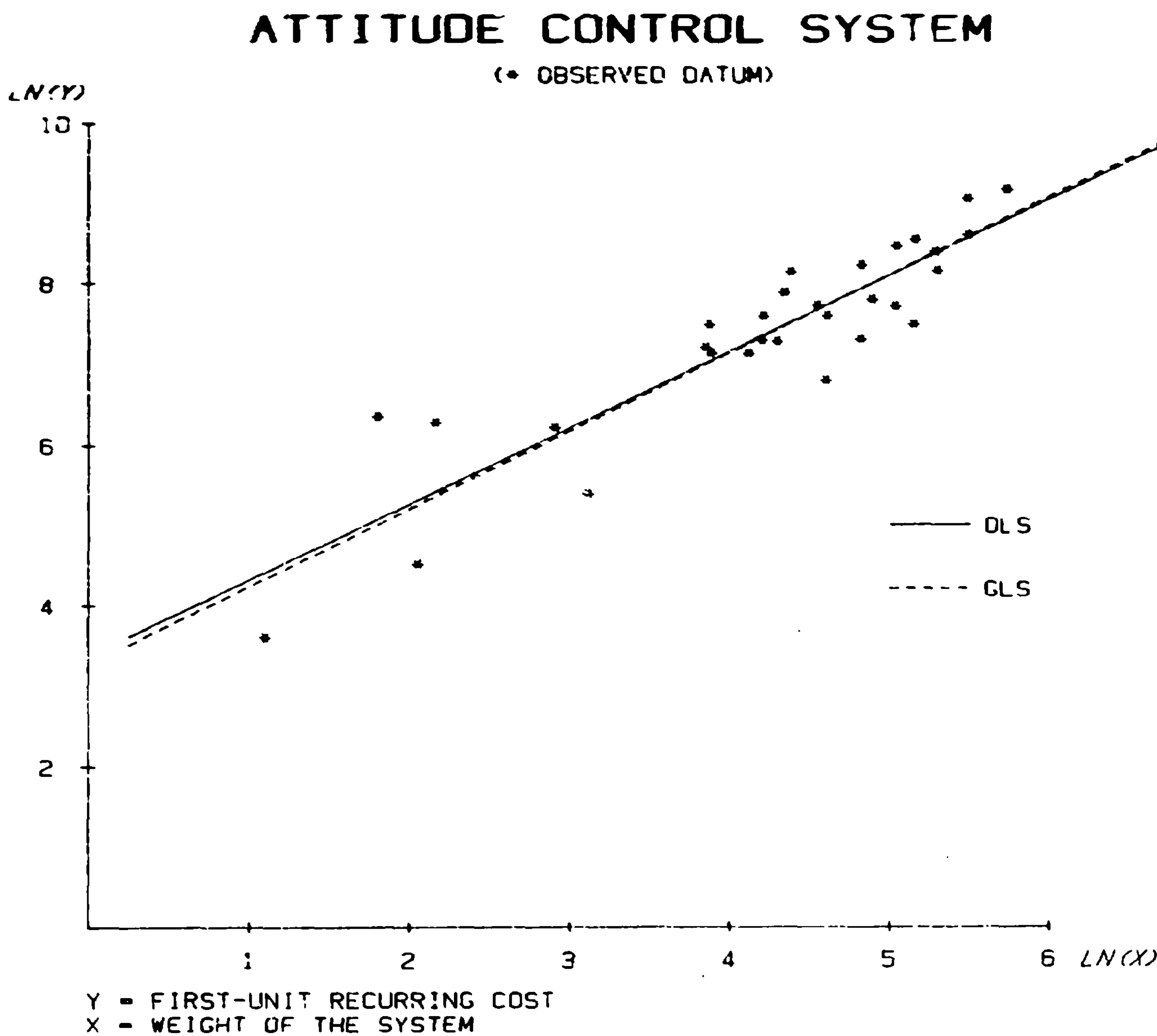
ATTITUDE CONTROL SYSTEM
(* OBSERVED DATUM)
LN(Y)
10
8
6
4
2
1
2
3
4
5
6
LN(X)
OLS
GLS
Y = FIRST-UNIT RECURRING COST
X = WEIGHT OF THE SYSTEM

FIGURE 2

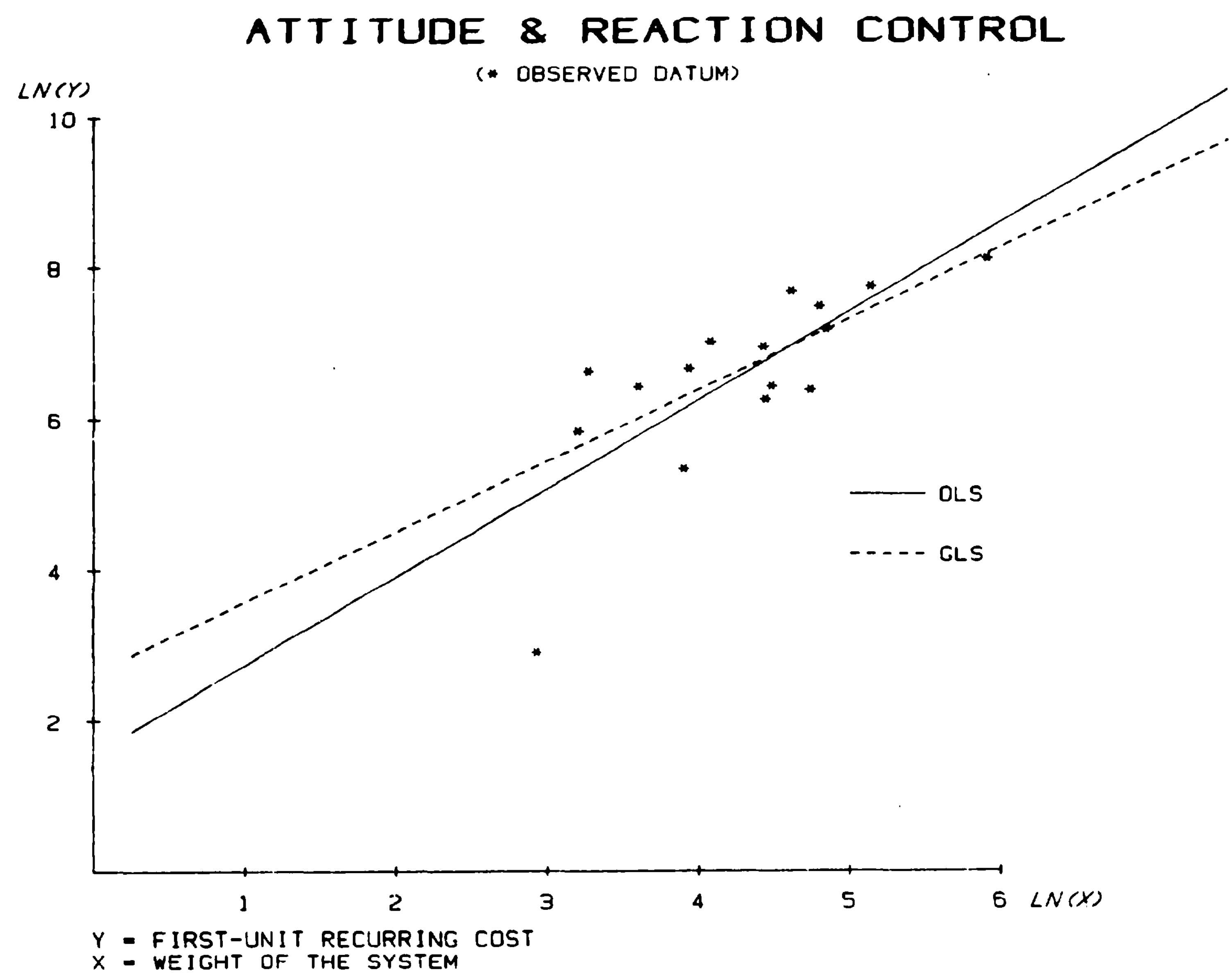
ATTITUDE & REACTION CONTROL
(* OBSERVED DATUM)
LN(Y)
10
8
6
4
2
1
2
3
4
5
6
LN(X)
OLS
GLS
Y = FIRST-UNIT RECURRING COST
X = WEIGHT OF THE SYSTEM

FIGURE 3

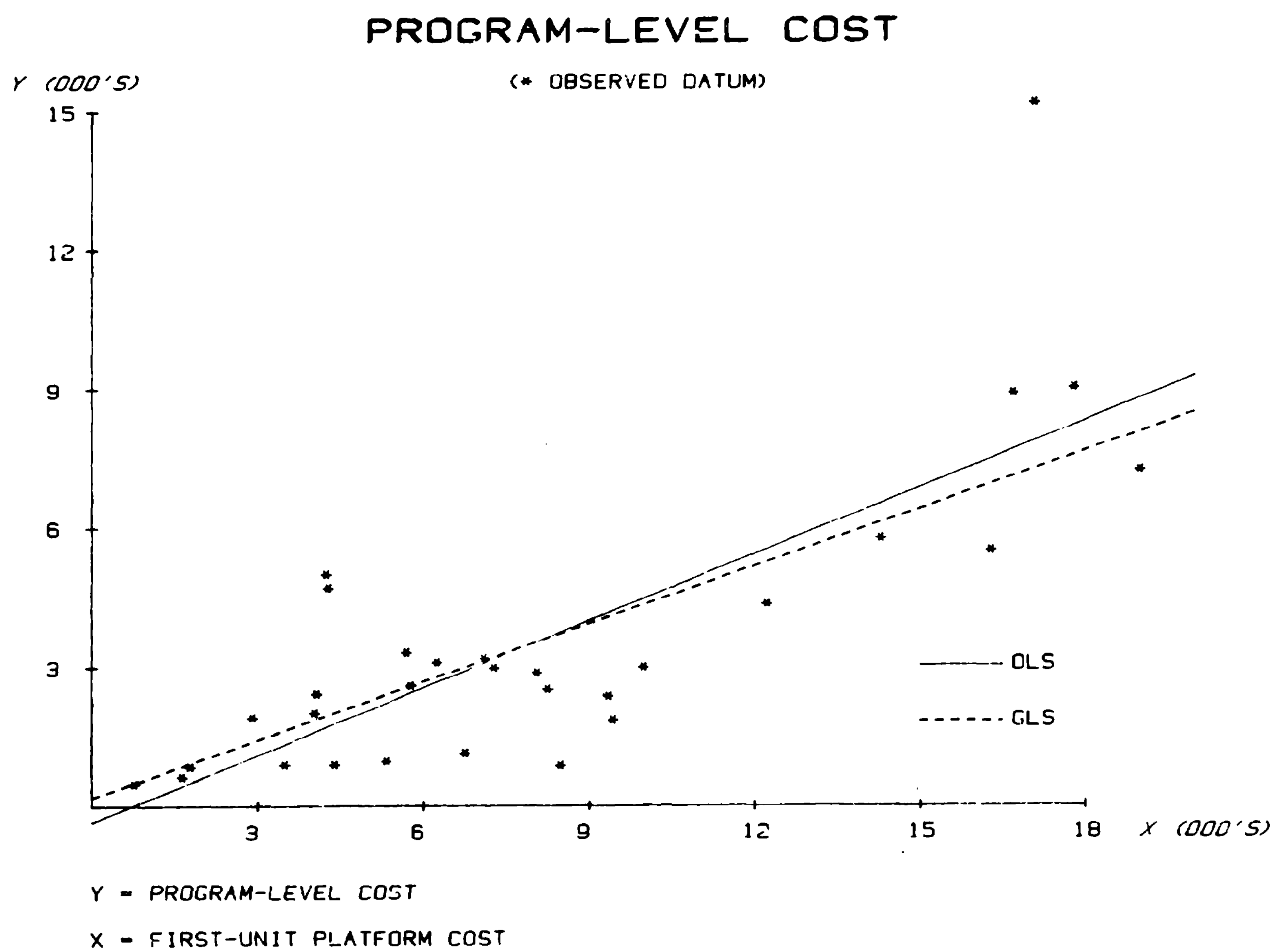
PROGRAM-LEVEL COST
(* OBSERVED DATUM)
Y (000'S)
15
12
9
6
3
3
6
9
12
15
18
X (000'S)
OLS
GLS
Y - PROGRAM-LEVEL COST
X - FIRST-UNIT PLATFORM COST

FIGURE 4

III. THE GLS REMEDY

A. General

The brute and blind mechanical nature of Ordinary Least-Squares (OLS) gives excessive weight to observations on Y that are associated with large error variances. In the Attitude and Reaction Control CER (Figure 3), for example, the position of the least-squares line is governed inordinately by those data points that are most spread out, i.e., by those associated with relatively lightweight systems. OLS estimates of regression parameters are consequently no longer of minimum variance, although they do remain unbiased.[3]

Generalized Least Squares (GLS) is a statistical technique which alleviates the problem of heteroskedasticity in a regression equation. It adjusts observations on Y and X so that the variance of the equation's error term is once again constant, as Appendix 2 details.

B. GLS Estimates

GLS estimates of the parameters in the three CER's are compared to their OLS counterparts in Table 2. Differences are small for the first CER but substantial for the remaining two.

In the Attitude and Reaction Control equation, as Figure 3 shows, the OLS regression line seems a little too steep, with its position inordinately influenced by the outlier in the southwest quadrant of the chart. And in the Program-Level Cost CER, as Figure 4 shows, the ordinary least-squares line again seems too steep, with the northeastern outlier appearing particularly pernicious.[4]

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[3] See Kmenta [2] for a detailed explanation.

[4] Excluding these recalcitrant data points from their respective samples and then re-estimating using OLS gives values close to those obtained by GLS in the case of the second CER, but not the first. In the Attitude and Reaction Control equation, the revised OLS line is flatter than the GLS line by a fair margin.

In either event, however, GLS is preferred. It uses all sample data, and has optimal statistical properties. The outliers, in other words, are partly but not fully to blame for the bugaboo of heteroskedastic disturbances. Indeed, they're symptomatic of the problem.

TABLE 2

COMPARISON OF OLS AND GLS ESTIMATES
(t-statistics in parentheses)

CER/Summary Statistics	OLS Estimates $\ell n \alpha$	OLS Estimates β	GLS Estimates $\ell n \alpha$	GLS Estimates β
ATTITUDE CONTROL	3.370	0.945	3.265	0.967
	(9.055)	(11.090)	(6.073)	(8.633)
R-Squared	0.814		0.997	
F-Statistic	122.882		4847.543	
DW Statistic	2.711		2.390	
ATTITUDE & REACTION CONTROL	1.559	1.172	2.630	0.940
	(1.308)	(4.261)	(3.097)	(5.528)
R-Squared	0.565		0.996	
R-Statistic	18.159		1873.922	
DW Statistic	1.761		2.584	
PROGRAM-LEVEL COST	-338.815*	0.480	184.619*	0.414
	(-0.493)	(6.681)	(0.511)	(6.557)
R-Squared	0.615		0.792	
F-Statistic	44.635		53.449	
DW Statistic	1.242		1.208	

*These are estimates of α rather than $\ell n \alpha$

NOTES: 1. Summary statistics and t-values for GLS estimation are from the transformed GLS equation, i.e., the equation with values of Y and X adjusted to yield an error term with constant variance (see Appendix 2).

2. Further, the mechanics of GLS require that the Y-intercept of the transformed equation be restricted to zero. Hence, each R-Squared statistic shown above is computed about a mean of zero.

3. Comparison of OLS and GLS R-Squared's or F's is invalid since they are based on regressions using two different dependent variables.

C. Cost Comparison

Cost estimates based on GLS are compared to their OLS counterparts in Table 3 for a quartet of sample obervations on each explanatory variable, i.e., for the mean of X, for ±50% of the mean, and for 300% above the mean. This latter percentage is included to capture the frequent case where a cost estimate is needed for a proposed piece of hardware whose weight lies outside the range of the weights of those spacecraft subsystems used to estimate the CER.

GLS and OLS predictions differ the most for observations wide of the mean, with the percentage delta increasing in absolute value as X becomes relatively small or relatively large. This isn't surprising since the GLS and OLS regression lines intersect near the average value of X in all three CER's, as Figures 2 through 4 show.

TABLE 3

COMPARISON OF GLS AND OLS COST ESTIMATES
(Costs are in thousands of FY79 constant dollars)*

CER	Value of X	Predicted Cost GLS	OLS	Delta	%Delta
ATTITUDE CONTROL					
0.5*Mean	52.5	$1206.0	$1227.8	$21.8	1.8%
Mean	105.0	$2357.6	$2363.7	$ 6.1	0.3%
1.5*Mean	157.6	$3491.5	$3469.5	-$22.0	-0.6%
4.0*Mean	420.0	$9008.5	$8760.8	-$247.7	-2.7%
ATTITUDE & REACTION CONTROL					
0.5*Mean	47.6	$ 523.8	$ 439.8	-$84.0	-16.0%
Mean	95.3	$1005.9	$ 992.1	-$13.8	-1.4%
1.5*Mean	142.9	$1472.1	$1595.0	$122.9	8.3%
4.0*Mean	381.2	$3702.4	$5037.1	$1334.7	36.0%
PROGRAM-LEVEL COST					
0.5*Mean	4046.3	$1859.8	$1603.4	-$256.4	-13.8%
Mean	8092.7	$3535.0	$3545.7	$10.7	0.3%
1.5*Mean	12139.1	$5210.2	$5488.0	$277.8	5.3%
4.0*Mean	32370.8	$13586.1	$15199.2	$1613.1	11.9%

* All values are in unlogged form.

IV. CONCLUSION

A. Summary

Sixteen CER's of the Unmanned Spacecraft Cost Model were tested for non-constant error variances, or heteroskedasticity. Based on Park's two-tail t-test, the null hypothesis of homoskedasticity was rejected in three cases:

- Attitude Control
- Attitude and Reaction Control
- Program-Level Cost

Generalized Least Squares (GLS) was invoked to provide best, linear, unbiased (BLU) estimation. Differences between GLS and OLS estimates of regression-equation parameters were profound in the last two CER's.

B. Recommendations

Based on the foregoing analysis, this study recommends

1. Using GLS instead of OLS when heteroskedastic disturbances are suspected

2. Using observations on spacecraft unit costs from <u>outside</u> current NCD-5 samples to compare the predictive accuracy

of the GLS and OLS estimators of the above three CER's.

APPENDIX 1

PARK'S TEST FOR HETEROSKEDASTICITY

A simple linear equation of the spacecraft model is

(1) $Y_i = \alpha + \beta X_i + u_i \quad (i = 1, 2, \ldots, N)$, where

Y = first-unit hardware cost

X = hardware weight

u = a randomly distributed error term.

Further, α and β are population parameters to be estimated, and N is the number of spacecraft in the sample.

To test for heteroskedasticity, Park [3] proposes using

(2) $\mathrm{Var}(u_i) = \delta X_i^{\gamma} e^{\varepsilon_i}$, where

δ = an unknown constant

γ = a population parameter measuring degree of heteroskedasticity

$\mathrm{Var}(u_i)$ = the variance of u_i in equation (1)

ε_i = a well-behaved random error term.

For values of γ statistically different from zero, the error term in equation (1) will be heteroskedastic since Var (u_i) will change as X_i changes.

To estimate γ, the values $\hat{u}_i^2$ from OLS estimation of equation (1) are used as proxies for observations on $\mathrm{Var}(u_i)$ in equation (2). Taking logs,

$$(3) \quad \ln(\hat{u}_i^2) = \ln\delta + \gamma\ln(X_i) + \varepsilon_i$$

with the significance of γ examined using the two-tailed t-test.

APPENDIX 2

GENERALIZED LEAST SQUARES

Using results from Park's test of Appendix 1,

(4) $\quad \mathrm{Var}(\hat{u}_i) = \hat{\delta} X_i^{\hat{\gamma}}$, or in words,

the variance of the random error term in equation (1) is related to the value of the explanatory variable, X_i.

Generalized Least Squares (GLS) is implemented by

- Dividing equation (1) by $X_i^{\hat{\gamma}/2}$, denoted w_i for simplicity,

$$Y_i/w_i = \alpha/w_i + \beta X_i/w_i + \underbrace{u_i/w_i}_{\text{constant variance}}$$

- Estimating this equation using OLS, with the term $1/w_i$ regarded as a second explanatory variable, and with the Y-intercept restricted to zero.

Since the transformed error term is of constant variance, i.e.,

$$E(u_i/w_i)^2 = \mathrm{Var}(u_i)/w_i^2 = \delta \ ,$$

the Gauss-Markov theorem now applies, and least-squares estimates are best, linear, unbiased (BLU).

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[3] Park, R.E., "Estimation with Heteroskedastic Error Terms," Econometrica, Vol. 34, No. 4, October 1966, p. 888.

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